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Gain Enhancement in Electromagnetically Pumped Free Electron Laser With a Guide Magnetic Field

Ahmad OMAR, Sinan BİLİKMEN
Department of Physics, Middle East Technical University
06531, Ankara-TURKEY

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Abstract

An electromagnetic wiggler field is used as a pump in this work. A guide magnetic field is added to this configuration, since it is necessary in order to confine the beam against the effects of self field. A small amplitude backward electromagnetic wave is introduced as an additional perturbed electromagnetic wave, and first order perturbation analysis is applied. A small signal gain formula is derived for this system. Numerical calculations shows that this backward radiation field contributes for gain enhancement of the system when it approaches orbital instability. This enhancement of the gain is found within a new range of both the wiggler field amplitude and the perturbed frequency.

1. Introduction

In recent years, a great departure from conventional lasers to new interesting and greatly desirable lasers have been achieved after the developments of the theory and experiment of FREE ELECTRON LASERS (FEL). FEL is remarkable in its various applications, and for diagnostics in atomic, molecular and solid state physics. Its applications extend to biology, medicine, chemistry, industry and defense [1,2,3].

In this study we want to exploit an electromagnetic wave as a pump instead of the static magnetic field. The development of two-stage free electron lasers facilitate the implementation of the idea of the electromagnetic pump. In this laser a long-wavelength wave from a low-energy free electron laser is used as the wiggler for a short-wavelength FEL using the same, low energy electron beam [2,4].

Carmel et al., demonstrated the first operation of a two-stage backward-wave FEL[5]. Backward waves oscillator functions as the pump wave in his study. Freund et al.[6], studied the interaction between a relativistic electron beam and an electromagnetic wiggler in the second stage. His work represents an introduction to a more comprehensive analysis of the linear gain for the same system. It is found that in using electromagnetic pump $\vec{\nabla} \times \vec{B}_w$ is balanced by the displacement current, allowing, in principle, propagation of purely one-dimensional waves in large interaction volumes[3].

Recently, H. Freund et al.[6,7] used a configuration which consists of a uniform axial-guide magnetic field, and a backwards propagating electromagnetic wave. Linear gain is calculated for this configuration which includes an additional small-amplitude plane electromagnetic wave propagating antiparallel to the large amplitude electromagnetic wiggler and parallel to the axial-guide field and the direction of electron flow.

L.Friedland et al.[3] derived the small signal gain in the case of a configuration consisting of forward electromagnetic pumped FEL with a guide magnetic field which include an additional small amplitude plane electromagnetic wave propagating parallel to both the electromagnetic pump and axial-guide field and to the direction of electron flow.

H.Freund et. al [6,7] concluded that his approach concerning the gain is more convenient than that which was done by Friedland. But we think that this is incorrect. Because the different assumptions for the two configurations lead to different results.

This work is an extension to that done by L.Friedland. In section 2.1, we will review the dynamics of the relativistic electron beams in combined guide and electromagnetic pump field. Also, the possibilities for a helical steady state equilibria is discussed. In section 2.2, we go through stability analysis, and the stability of the helical equilibria is discussed. Section 3, concerned with deriving a formula for the small signal gain in FEL in the case of a small-amplitude backwards vacuum electromagnetic field, which is used as an additional perturbed electromagnetic field. In our calculations we follow the same method used by L.Friedland et al.[3] in calculating the gain formula for the same pump and guide fields, but with different perturbed electromagnetic fields. Section 4 will concentrate on numerical examples for the performance of different FEL amplifiers, and analysis of the results physically. Results and conclusions and comparison with previous results are presented. Suggestions for future investigations are reported.

2. Equations of Motion

2.1. Steady state solutions

In this section we will deal with a cold relativistic electron beam propagating in an electromagnetic field which is derived from the vector potential [3]

$$\vec{A}_0 = -A_0[Cos(k_0 z - \omega_0 t)\hat{e}_x + Sin(k_0 z - \omega_0 t)\hat{e}_y] \, . \tag{1}$$

The electromagnetic field is given by

$$\vec{E}_0 = -\frac{1}{c}\frac{\partial \vec{A}_0}{\partial t}, \vec{B}_0 = \vec{\nabla} \times \vec{A}_0 + B_1 \hat{e}_z \, , \tag{2}$$

where B_1 =const.(the guide magnetic field)[9,10]. The final form of the electromagnetic field is

$$\vec{E}_0 = -\frac{\omega_0 A_0}{c}[Sin(k_0 z - \omega_0 t)\hat{e}_x - Cos(k_0 z - \omega_0 t)\hat{e}_y]$$

$$\vec{B}_0 = k_0 A_0 [Cos(k_0 z - \omega_0 t)\hat{e}_x + Sin(k_0 z - \omega_0 t)\hat{e}_y] + B_1 \hat{e}_z \, . \tag{3}$$

It is assumed that the density of the electron beam is low and does not influence $\vec{E}_0$ and $\vec{B}_0$.

At this stage, we will study the dynamics of the electron beam using the momentum equation

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right](m\gamma\vec{v}) = -e\left[\vec{E}_0 + \frac{\vec{v}\times\vec{B}_0}{c}\right] \tag{4}$$

This equation is rewritten as follows

$$\left[\frac{\partial}{\partial \tau} + \vec{U}.\vec{\nabla}\right](\gamma\vec{U} - \vec{\alpha}) = \Omega\hat{e}_z \times \vec{u} - (\vec{\nabla}\vec{\alpha}).\vec{U}, \tag{5}$$

where

$$\hat{e}_1 = -\hat{e}_x Sin\phi + \hat{e}_y Cos\phi \tag{6}$$

$$\hat{e}_2 = -\hat{e}_x Cos\phi - \hat{e}_y Sin\phi \tag{7}$$

$$\hat{e}_3 = \hat{e}_z \tag{8}$$

and $\phi = k_0 z - \omega_0 t$, $\vec{\alpha} = \frac{eA_0}{mc^2}\hat{e}_2 = \alpha_0 \hat{e}_2$, $\tau = ct$, $\vec{U} = \frac{\vec{v}}{c}$, $\Omega = \frac{eB_1}{mc^2}$, $\omega_0' = \frac{\omega_0}{c}$. The coefficients of the identical base vectors can be written separately as follows

$$\left[\frac{\partial}{\partial \tau} + u_3\frac{\partial}{\partial z}\right](\gamma u_1) + (\omega_0' - u_3)(\gamma u_2 - \alpha_0) = -\Omega u_2 \tag{9}$$

$$\left[\frac{\partial}{\partial \tau} + u_3\frac{\partial}{\partial z}\right](\gamma u_2) - (\omega_0' - k_0 u_3)(\gamma u_1) = \Omega u_1 \tag{10}$$

$$\left[\frac{\partial}{\partial \tau} + u_3\frac{\partial}{\partial z}\right](\gamma u_3) = \alpha_0 k_0 u_1. \tag{11}$$

Note that Eqs.(9)-(11) have a particular solution $u_1 = 0$, $u_2 = u_{20} = Const$, $\gamma = \gamma_0 = Const.$. Substituting this value for the particular solution into Eq.(9) we find

$$u_{20} = \frac{\frac{(u_{30}k_0 - \omega_0')\alpha_0}{\gamma_0}}{(u_{30}k_0 - \omega_0') - \frac{\Omega}{\gamma_0}} \tag{12}$$

But we know that γ (the relativistic factor) is defined as

$$\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$$

Define $\frac{\vec{v}}{c} = \vec{u}$, $\vec{u} = u_1\hat{e}_1 + u_2\hat{e}_2 + u_3\hat{e}_3$. This implies that

$$\frac{1}{\gamma_0^2} = 1 - u_{20}^2 - u_{30}^2$$

Equations (12) and (14) represent four possible steady-state [illegible] Ω, k_0 and ω_0'. It is found that the critical values for the wiggler [illegible] calculated from the formula

$$\frac{du_{30}}{d\Omega} = -\frac{{\alpha_0}^2(k_0u_{30} - \omega_0')^2}{au_{30}{\gamma_0}^3\mu_0^2}$$

where

$$a{\mu_0}^2 = a^3 - \frac{{\alpha_0}^2{k_0}^2\Omega}{\gamma_0^3}[1 - \frac{v_p}{u_{30}}], \quad (16)$$

$$a = [k_0u_{30} - \omega_0'] - \frac{\Omega}{\gamma_0}. \quad (17)$$

and $v_p = \frac{\omega_0'}{k_0}$.

2.2. Stability Condition

In this section we will use the perturbed steady-state helical equilibrium. Then we obtain a homogeneous second order differential equation, which will be very helpful in studying the stability of the steady-state helical trajectories.

We shall follow A. Goldring and L.Friedland[3], to obtain this differential equation. We begin our analysis by writing the solutions of Eqs.(9)-(11) of the previous section in the form

$$u_i = u_{i0} + \omega_i, \; i = 1, 2, 3 \quad (18)$$

where u_i's are constants or zero, while ω_i's are assumed to be small perturbations to an adequate degree so they will obey linearized equations. The equation representing the perturbation in energy is

$$\gamma = \gamma_0 + g \quad (19)$$

where g is a small perturbation in energy. Substituting Eqs.(18) and (19) into Eqs.(9)-(11), then rearranging terms after taking first order perturbation terms and using the energy equation

$$mc^2\dot{\gamma} = -e\vec{E}_0 \cdot \vec{v} \quad (20)$$

we found

$$\ddot{\omega_1} + {\mu_s}^2 \omega_1 = 0 \tag{21}$$

where

$$a\mu_s^2 = a^3 - \frac{k_0^2 {\alpha_0}^2 \Omega}{\gamma_0^3}[1 - v_p^2] \tag{22}$$

We note that the necessary condition for stability of the steady-state helical trajectory is ${\mu_s}^2 > 0$.

3. Small Signal Gain

3.1. The Method

L.Friedland succeeded in calculating the gain in different FEL configurations using this recent method. The general formula which was derived by L.Friedland is written as follows[3,8]

$$\Gamma = -\frac{(\hbar\omega)^2 N_e}{Q} \frac{d}{d\epsilon}(u_{30} < n >_{av}) \tag{23}$$

where

$$Q = \frac{E^2}{8\pi} \tag{24}$$

E is the perturbed electromagnetic wave amplitude, Q is the time-averaged energy density of the perturbing electromagnetic wave, N_e is the particle density of the electron beam.

$$\epsilon = mc^2 \gamma_0(t) \tag{25}$$

where ϵ is the average number of quanta $\hbar\omega$ absorbed by an electron in a single pass through the interaction region.
We find $< n >_{av}$ from

$$< n >_{av} = \frac{< s^2 >_{av}}{2(\hbar\omega)^2} \tag{26}$$

where s is given by

$$s = -ec \int_{t1}^{t2} \vec{u}(t') \cdot (\vec{E}_0(z(t'), t') + \vec{E}(z(t'), t'))dt' \tag{27}$$

We note that the previous integration is with respect to the time t' along the electron trajectory. The moments at which the electron enters and leaves the interaction region are represented by the symbols t_1 and t_2.

Now we consider the case in which the electrons move in one of the steady- state helical trajectories, discussed in section 2. In this case we assume no perturbed electromagnetic waves. So we can write $\vec{u}$ as $\vec{u} = \vec{u_0} + \vec{\omega_i}$, so that correct to first order in E

$$s = -ec\left[\int_{t1}^{t2} (\vec{u_0} \cdot \vec{E})\, dt + \int_{t1}^{t2} (\vec{\omega_i} \cdot \vec{E_0})\, dt\right] \tag{28}$$

3.2. Gain Calculation

In this section we will apply the method stated in Sec.2.1 to calculate the small signal gain when a small amplitude backwards electromagnetic wave is considered as a perturbed radiation field:
This electromagnetic field is written as follows[6,7]

$$\vec{E} = E[-\hat{e}_x Sin(kz + \omega t) + \hat{e}_y Cos(kz + \omega t)] \tag{29}$$
$$\vec{B} = E[\hat{e}_x Cos(kz + \omega t) + \hat{e}_y Sin(kz + \omega t)] \tag{30}$$

Using the following transformations and definitions
$t \to t'$, $\tau' = ct$, $\omega' = \frac{\omega}{c}$, $\omega_0{}' = \frac{\omega_0}{c}$. and

$$\beta = (k - k_0)z + (\omega' + \omega_0{}') \tag{31}$$
$$\psi = kz_0 + \omega'\tau_1 + \frac{\omega' T}{2} \tag{32}$$
$$\phi_1 = -k_0 z_0 + \omega_0'\tau_1 + \frac{\omega_0' T}{2} \tag{33}$$

where $T = \tau_2 - \tau_1$, $\tau_1 < \tau' < \tau_2$, $0 < \tau'' < \tau_2$, $\tau' = \tau'' + \tau_1 + \frac{T}{2}$, $z = z_0 + u_{30}\tau''$
Equations (29)-(30) become

$$\vec{E} = E[\hat{e}_1 Cos(\beta\tau'' + \psi + \phi_1) + \hat{e}_2 Sin(\beta\tau'' + \psi + \phi_1)] \tag{34}$$
$$\vec{B} = E[\hat{e}_1 Sin(\beta\tau'' + \psi + \phi_1) - \hat{e}_2 Cos(\beta\tau'' + \psi + \phi_1)] \tag{35}$$

When we consider the case for $\beta \simeq 0$, we find

$$\omega' = \frac{k_0 u_{30} - \omega_0'}{1 + u_{30}} \tag{36}$$

Since $u_{20} << u_{30}$, the last equation can be rewritten as follows

$$\omega' = \gamma_0{}^2(1 - u_{30})(k_0 u_{30} - \omega_0') \tag{37}$$

Returning back to Eq.(28) in this section, we want to evaluate the first integral using the new electric field.

$$I_1 = -e\int_{-\frac{T}{2}}^{\frac{T}{2}} (\vec{u}_0 \cdot \vec{E}) d\tau'' \tag{38}$$

where
$-\frac{T}{2} < t' < \frac{T}{2}$, $\tau' = ct'$, $d\tau' = d\tau''$
The previous equation can be simplified as follows

$$I_1 = -\frac{2eu_{20}ESin(\frac{\beta T}{2})Sin(\psi + \phi_1)}{\beta} \tag{39}$$

In order to find the integrand of the second integral in Eq.(28) for ω_1 due to the amplified wave, we go back to the momentum equations (Eqs.(9-11)) and add new electromagnetic interaction terms to it.

After tedious algebra we found a second order differential equation in ω_1

$$\ddot{\omega_1} + {\mu_s}^2\omega_1 = N\epsilon_2 \tag{40}$$

where

$$N = \frac{1}{\gamma_0}\left[(u_{30}+1)(a-\beta) - \frac{\Omega}{\gamma_0}\frac{(1+v_p)}{(u_{30}-v_p)}u_{20}^2\right] \tag{41}$$

and

$$\epsilon_2 = -\epsilon_0 Sin(\beta\tau'' + \psi + \phi_1), \qquad \epsilon_0 = \frac{eE}{mc^2}$$

We used the method of undetermined coefficients to solve this inhomogeneous second order differential equation. We considered the following initial conditions in this analysis

$$\omega_1(\tau_1) = 0, \quad \dot{\omega_1}(\tau_1) = r_1.$$

The general solution for Eq.(40) is given approximately ($\beta \to 0$, and $\mu_s^2 >> \beta^2$) by the following equation

$$\omega_1(\tau'') \simeq -\frac{N\epsilon_0}{\mu_s^2} Sin(\beta\tau'' + \psi + \phi_1) \tag{42}$$

But we know that

$$I_2 = -\frac{eA_0\omega_0}{c}\int_{-\frac{T}{2}}^{\frac{T}{2}} \omega_1 d\tau'' = \frac{e^2A_0\omega_0 NE}{mc^3\mu_s^2}\int_{-\frac{T}{2}}^{\frac{T}{2}} Sin(\beta\tau'' + \psi + \phi_1)d\tau'' \tag{43}$$

The previous equation can be rewritten as follows

$$I_2 = \frac{2e^2 A_0 \omega_0 N E Sin(\frac{\beta T}{2}) Sin(\psi + \phi_1)}{\beta} \tag{44}$$

Introducing the following parameters $\theta = \frac{\beta T}{2}$, $T = \frac{L}{u_{30}}$
so

$$\beta = \frac{2u_{30}\theta}{L} \tag{45}$$

Equation (39) becomes

$$I_1 = -\frac{eu_{20}EL}{u_{30}} Sin(\psi + \phi_1)\frac{Sin\theta}{\theta} \tag{46}$$

Equation (44) becomes

$$I_2 = \frac{\alpha_0 \omega_0 N e E L}{c\mu_s^2 u_{30}} Sin(\psi + \phi_1)\frac{Sin\theta}{\theta} \tag{47}$$

Since $s = I_1 + I_2$, and $E^2 = 8\pi Q$, $< s^2 >_{av}$ can be written as

$$< s^2 >_{av} = 4\pi Q e^2 L^2 \left[-u_{20} + \frac{\alpha_0 \omega_0 N}{c\mu_s^2}\right]^2 \left[\frac{Sin\theta}{\theta}\right]^2 \tag{48}$$

and

$$< n >_{av} = \frac{2\pi Q e^2 L^2}{(\hbar\omega)^2 u_{30}^2} \left[-u_{20} + \frac{\alpha_0 \omega_0 N}{c\mu_s^2}\right]^2 F(\theta) \tag{49}$$

where

$$F(\theta) = \left[\frac{Sin\theta}{\theta}\right]^2 \tag{50}$$

So we can write the gain formula as follows

$$\Gamma \simeq -\frac{2\pi N_e e^2 L^2}{u_{30}} \left[-u_{20} + \frac{\alpha_0 \omega_0 N}{c\mu_s^2}\right]^2 \frac{dF(\theta)}{d\epsilon} \tag{51}$$

But

$$\frac{dF(\theta)}{d\epsilon} = \frac{dF}{d\theta}\frac{d\theta}{d\epsilon} \tag{52}$$

and

$$\frac{d\theta}{d\epsilon} = -\frac{(\omega' + \omega_0')L}{2u_{30}^2}\frac{du_{30}}{d\epsilon} \tag{53}$$

Equation (51) can be simplified as follows

$$\Gamma = \frac{\pi N_e e^2 L^3}{u_{30}^3}\left[-u_{20} + \frac{\alpha_0\omega_0 N}{c\mu_s^2}\right]^2 \frac{dF}{d\theta}(\omega' + \omega_0')\frac{du_{30}}{d\epsilon} \tag{54}$$

Combining Eq.(14) and Eq.(25)together and differentiating the resulting equation with respect to ϵ, we found

$$\frac{du_{30}}{d\epsilon} = \frac{a^3 + \alpha_0^2 k_0^3(u_{30} - v_p)^3}{mc^2 u_{30}\gamma_0^3 a\mu_0^2} \tag{55}$$

Now we substitute Eq.(55) in Eq.(54), the gain formula for this system becomes

$$\Gamma = \frac{\pi N_e^2 L^3(\omega' + \omega_0')}{mc^2\gamma_0^3 u_{30}^4\mu_0^2 a}[a^3 + k_0^3(u_{30} - v_p)^3\alpha_0^2]\left[-u_{20} + \frac{\alpha_0\omega_0 N}{c\mu_s^2}\right]^2 \frac{dF(\theta)}{d\theta}. \tag{56}$$

4. Results and Conclusions

The gain formula for electromagnetically pumped FEL with a guide magnetic field was derived by using an additional small amplitude backward plane vacuum electromagnetic wave as a perturbed radiation field.

The contribution of a small amplitude backwards radiation field for gain enhancement of the system will be discussed explicitly, below.

In section 3, we calculated the gain by introducing a small backwards electromagnetic wave as an additional perturbed radiation field. Comparison between the numerical results for two configurations (L.Fiedland et al. and ours) is reported.
However, the following set of parameters $\gamma_0 = 3$, $L = 100cm$, $\mid u_\perp \mid = 0.1$, and the frequency of the amplified radiation was $\omega' = 180cm^{-1}(f \simeq 10^{12}Hz)$ have been choosen. We want to acknowledge numerical mistakes done by L.Friedland et al. We noticed that they choose $\omega^{'} = 180cm^{-1}$, and discussed three situations for the phase velocity $v_p = 0, 0.85, 1.3$ respectively. In reviewing their calculations we found that $k_0 = 12$, 6.3, and $5cm^{-1}$ does not satisfy these values for v_p's, the correct values are $v_p = 1.9$, 2.7, and 3.2, but if we choose $k_0 = -12, -6.3, -5cm^{-1}$, the resulted phase velocities of the pump are $v_p = 0$, -0.85, and -1.3 respectively. This implies that the pump wave propagates opposite to electron beam motion, which contradicts with their numerical assumptions.
Also, we want to acknowledge the corrections to Eqns. (39)-(41) in Ref.[3] , the correct forms are given as follows

$$\epsilon_1 = \frac{eE}{2mc^2}Sin(\beta_1\tau^{''} + \psi - \phi_1) \tag{57}$$

$$\epsilon_2 = -\frac{eE}{2mc^2}Cos(\beta_1\tau^{''} + \psi - \phi_1) \tag{58}$$

and

$$A = \frac{1}{\gamma_0}\left[-(1-u_{30})(a-\beta_1) + \frac{\Omega}{\gamma_0}u_{20}^2(1-v_p)/(u_{30}-v_p)\right] \tag{59}$$

For their configuration, the relation between $\omega^{'}$ and $\omega_0^{'}$ is determined by[3]

$$\omega^{'} = 2\gamma_0^2(\omega_0^{'} - k_0 u_{30}) \tag{60}$$

which is derived from

$$\beta_1 = (k-k_0)u_{30} - (\omega^{'} - \omega_0^{'}) \tag{61}$$

under the assumption that $\beta_1 \simeq 0$ and $u_{20} << u_{30}$.
We repeated the calculations of Friedland's work after taking the corrections into considerations.
We adjusted the guide field so that $u_\perp = 0.1$. The normalized gain parameter $\eta = 10log_{10} \mid \frac{\Gamma}{N_e F'} \mid$ is plotted against the pump field strength α_0, and this was represented for the three previously mentioned configurations in Figure 1a. From these figures we note that there are significant gain enhancements at certain values of α_0, which satisfy the following analytical formulas

$$\alpha_{cr1} = \frac{\gamma_0}{u_{30}(\frac{u_{30}-v_p}{u_{20}^3} + \frac{1}{u_{20}})} \tag{62}$$

and

$$\alpha_{cr2} = \frac{\gamma_0(1-v_p^2)}{\frac{(u_{30}-v_p)^2}{u_{20}^3} + \frac{(1-v_p^2)}{u_{20}}} \tag{63}$$

Figure 1b show that these critical points corresponding to the conditions μ [illegible] $\to 0$, μ [illegible] $\to 0$, which from a physics point of view describe the case of enhanced induced response of the system in the vicinity of orbital instability. Figure 1a shows us that [illegible] the gain takes place at very low values of the pump field. As an [illegible] the critical $\alpha_0 = 1.42 \times 10^{-3}$ corresponds to the magnetic component of the pump of only $\sim 12 Gauss$. However, it differs from the value reported by Friedland [illegible] B [illegible] $18 Gauss$ for $v_p = 1.3$)

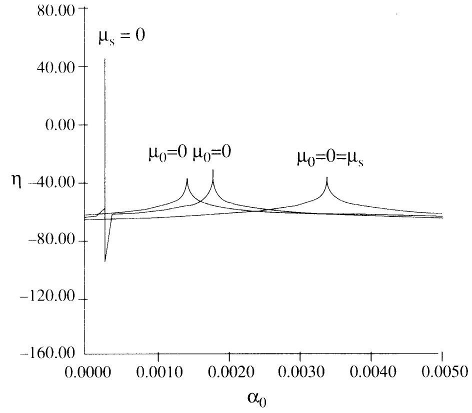


Figure 1a. Normalized gain η vs the strength of the pump field for three different FEL configurations ($v_p = 0, -0.85, -1.3$), $\gamma_0 = 3$, $L = 100cm$, $| u_\perp |= 0.1$, and $u_{30} = 0.937$, $\omega' = 180cm^{-1}$ [Friedland's work]

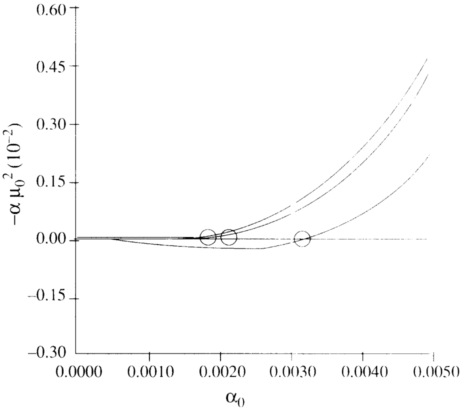


Figure 1b. Orbital stability parameter $a\mu_0^2$ vs α_0 for conditions of Fig.1a, ($v_p = 0, -0.85, -1.3$) [Friedland's work]

Now we will discuss our results which are illustrated in Fig.2a and Figures.2.1b, 2.2b, 2.3b for three different phase velocities of the pump. We used the same parameters for γ_0, L, and $u_\perp$. But this is not the case for the rest of the parameters and especially k_0 and ω_0', because there is another relation between them which is derived in section 3,

$$\beta = (k - k_0)u_{30} + (\omega' + \omega_0') \tag{64}$$

which can be rewritten as follows

$$\omega' = \gamma_0^2(1 - u_{30})(k_0 u_{30} - \omega_0') \tag{65}$$

for electron resonance condition $\beta \simeq 0$, and $u_{30} >> u_{20}$.
For the case of the conventional magnetostatically pumped FEL ($v_p = 0$, $k_0 = 12cm^{-1}$, $\omega_0' = 0$, $\omega' = 5.8cm^{-1}$), the critical $\alpha_0 \simeq 3.38 \times 10^{-3}$ corresponds to B_w of only 69 Gauss. For the case in which ($v_p = 0.35$, $k_0 = 20cm^{-1}$, $\omega_0' = 7cm^{-1}$, $\omega' = 6cm^{-1}$), we note that the critical $\alpha_0 = 5.35 \times 10^{-3}$ corresponds to the magnetic component of the pump of only $\sim 183 Gauss$.

The normalized gain parameter is plotted against the strength of the pump field for the three previously mentioned configurations. Figure 2a shows significant gain enhancements at certain values of α_0. These critical points -Figs.2.1b, 2.2b, 2.3b- corresponds to the conditions $\mu_0 \rightarrow 0$, $\mu_s \rightarrow 0$, which as we indicated previously in this section to describe the case of an enhanced induced response of the system in the vicinity of orbital instability.

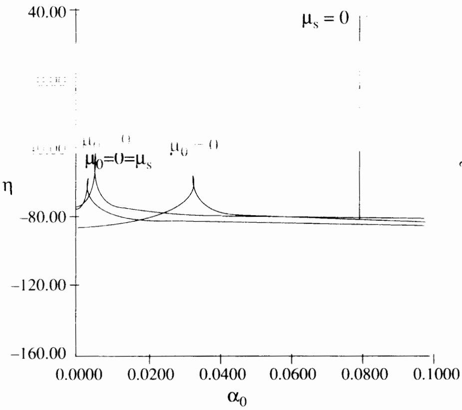


Figure 2a. Normalized gain η vs the strength of the pump field for three different FEL configurations ($v_p = 0, 0.35, 0.85$), $\gamma_0 = 3$, $L = 100cm$, $| u_\perp |= 0.1$, and $u_{30} = 0.937$, ($\omega' = 5.8cm^{-1}$, $6cm^{-1}$, $0.28cm^{-1}$)

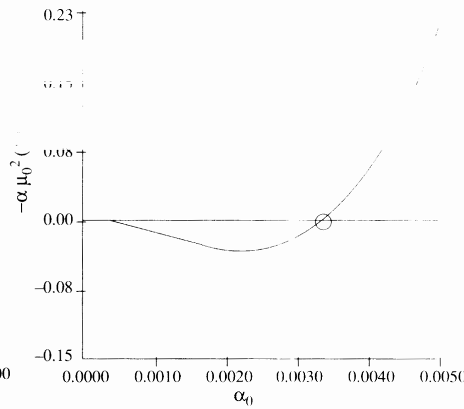


Figure 2.1b. Orbital stability parameter $a\mu_0^2$ vs α_0 for condition of Figure 2a. ($v_p = 0$, $\omega' = 5.8cm^{-1}$)

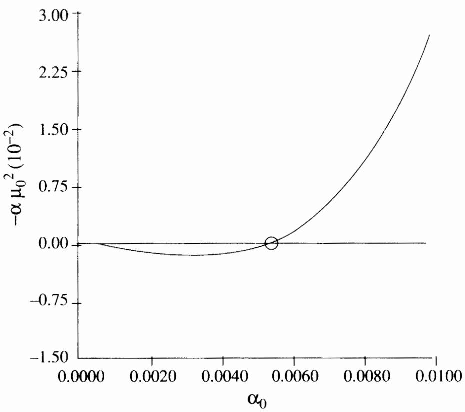


Figure 2.2b. Orbital stability parameter $a\mu_0^2$ vs α_0 for condition of Figure 2a.($v_p = 0.35$, $\omega' = 6cm^{-1}$)

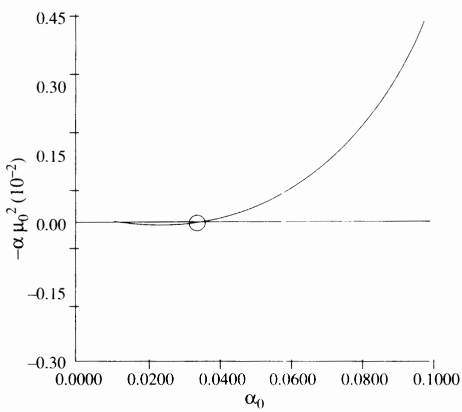


Figure 2.3b. Orbital stability parameter $a\mu_0^2$ vs α_0 for condition of Figure 2a. ($v_p = 0.85$, $\omega' = 0.28cm^{-1}$)

Previous results prove that the backward radiation field contribute for gain enhacement of the system under invesigations, at low – power strengths of the pump, but within new range of both the radiation field frequency, and the wiggler field amplitude compared to Friedland's results.

The relation between the radiated field frequency and the pump frequency at resonance for all possible configurations is plotted in Figs.3.1a, 3.2a. The corresponding relations between the radiated frequency and the phase velocity of the pump are plotted in Figs.3.1b, 3.2b respectively. It is clear that forward radiation can be achieved for $v_p > u_{30}$ and $v_p < 0$ for a forward and backward electromagnetic pump respectively. However, a backward radiation field is achieved when $0 < v_p < u_{30}$ for a forward pump.

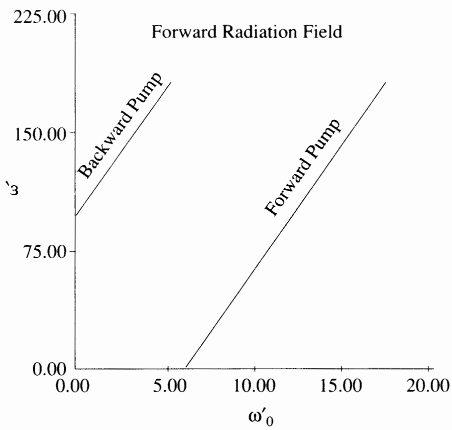


Figure 3.1a. The small amplitude vacuum forward radiation field frequency ω' vs the pump frequency ω_0'. ($k_0 = -6.3$, $k_0 = +6.3$), $u_{30} = 0.937$

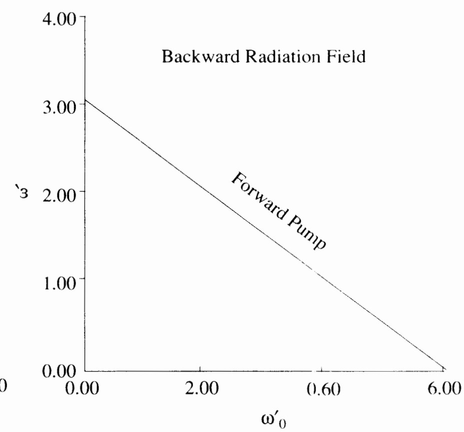
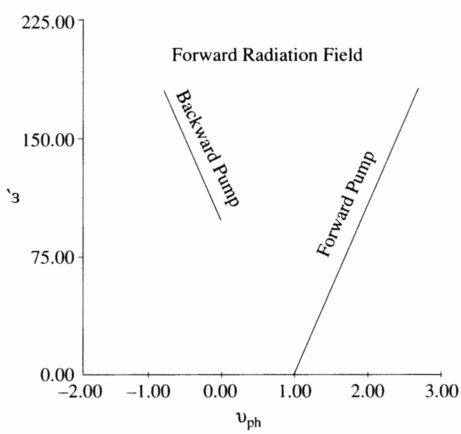


Figure 3.2a. The small amplitude vacuum backward radiation field frequency ω' vs the pump frequency ω_0'. ($k_0 = +6.3$), $u_{30} = 0.937$



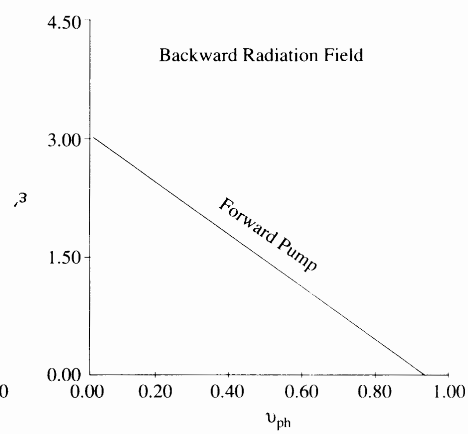


Figure 3.1b. The small amplitude vacuum forward radiation field frequency ω' vs the pump phase velocity v_p. ($k_0 = -6.3cm^{-1}$, $k_0 = +6.3cm^{-1}$), $u_{30} = 0.937$

Figure 3.2b. The small amplitude vacuum backward radiation field frequency ω' vs the pump phase velocity v_p. ($k_0 = 6.3cm^{-1}$), $u_{30} = 0.937$

These results show us that we can benefit from the different and wide range of frequencies that can be used by selecting the appropriate magnetic component of the pump to enhance the gain within the required radiation field frequency. Moreover, chaotic behaviour of phase space trajectories in electromagnetically pumped FEL is under investigations, and it will be the subject of a subsequent paper. This approach will clarify the integrable range of this system, which will be helpful in selecting the suitable parameters for efficient FEL operation.

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